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A NOTE ON A CLASS OF RINGS FOUND AS G_a-INVARIANTS FOR LOCALLY TRIVIAL ACTIONS ON NORMAL AFFINE VARIETIES

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ABSTRACT. This paper concerns the type of ring that can be realized as a ring of invariants for a locally trivial G_a-action on a normal, affine variety.

Results involving ideal-transforms and a counterexample to the problem of Zariski are utilized to achieve an example of a locally trivial action on a normal, affine variety of dimension 4 that has a nonfinitely generated ring of invariants. This would also yield yet another example of a G_a-action on an affine variety that can be written locally as a translation but does not admit an equivariant trivialization.

1. Introduction. The main result of this paper is to show that a class of rings can be realized as rings of invariants for additive group actions. The background is Hilbert's fourteenth problem, which asks the following: "Let k be an algebraically closed field and $x_1, \dots, x_n$ algebraically independent elements over k. Let L be a subfield of $k(x_1, \dots, x_n)$ containing k. Is the ring $L \cap k[x_1, \dots, x_n]$ finitely generated over k?" [**13**, p. 1]. Of particular interest is the case in which this intersection is the ring of invariants for a group action.

We first introduce some notation that will be used throughout the paper. Let k be an algebraically closed field of characteristic 0. We say that a k-algebra is affine if it is finitely generated as a k-algebra and that it is a normal domain if it is an integral domain that is integrally closed in its quotient field. Let $G_a = (k, +)$ denote the additive group on k. By an affine variety we will mean an irreducible, closed subset of k^n with respect to the Zariski topology. If $X \subseteq k^n$ is an affine variety, then when G_a act as automorphisms of the affine k-domain $k[X]$, it is well known that the associated k-homomorphism $k[X] \to k[X, t]$ is equivalent to a locally nilpotent k-derivation $D : k[X] \to k[X]$. That is, for a G_a-action $\sigma : G_a \times X \to X$, where for each $t \in G_a$, $\sigma_t \in \operatorname{Aut} X$,

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