

HYPERCYCLIC AND CHAOTIC CONVOLUTION OPERATORS ON CHÉBLI-TRIMÈCHE HYPERGROUPS

J.J. BETANCOR, J.D. BETANCOR AND J.M.R. MÉNDEZ

ABSTRACT. In this paper a universality property for Chébli-Trimèche convolution operators is proved. The results obtained extend prior analysis of the Fourier and Hankel transforms. We also investigate hypercyclic and chaotic convolution operators on Chébli-Trimèche hypergroups in some distribution spaces.

1. Introduction. In this paper we investigate new properties for the generalized Fourier transformation, also called Chébli-Trimèche transform, \mathcal{F} defined, when f is a suitable function defined on $(0, \infty)$, by

$$(\mathcal{F}(f)(\lambda) = \int_0^\infty \psi_\lambda(x) f(x) A(x) dx, \quad \lambda \geq 0,$$

where, for every $\lambda \geq 0$, ψ_λ represents the solution of the equation

$$(1.1) \quad \Delta \psi_\lambda(x) = (\lambda^2 + \rho^2) \psi_\lambda(x), \quad x > 0,$$

satisfying that

$$\psi_\lambda(0) = 1 \quad \text{and} \quad \frac{d}{dx} \psi_\lambda(0) = 0.$$

Here $\rho \geq 0$ and Δ denotes the differential operator

$$(1.2) \quad \Delta = -\frac{1}{A(x)} \frac{d}{dx} \left(A(x) \frac{d}{dx} \right),$$

where A is a real function on $[0, \infty)$ of the form $A(x) = x^{2\alpha+1} B(x)$, $\alpha > -1/2$, with B an even positive analytic function on \mathbf{R} satisfying $B(0) = 1$. We assume that A satisfies the following conditions

Research of the first author partially supported by DGICYT grant PB 97-1489 (Spain).

Research of the third author partially supported by DGICYT grant PB 97-1489 (Spain).

Received by the editors on April 6, 2001, and in revised form on July 12, 2002.