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SPACES $L_2(\lambda)$ OF A POSITIVE VECTOR MEASURE λ AND GENERALIZED FOURIER COEFFICIENTS

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ABSTRACT. Let L be a Banach lattice and consider a countably additive vector measure λ with values on L. Let $L_2(\lambda)$ be the Banach lattice of square integrable functions with respect to λ. In this paper we obtain several structure results for this space and the Fourier coefficients related to orthonormal sequences under the assumption that λ is positive. However, $L_2(\lambda)$ is not in general isomorphic to a Hilbert space. In fact, the norm of this space depends on the norm of L.

1. Introduction and basic results. Let L be a Banach lattice and let (Ω, Σ) be a measurable space. Let $\lambda : \Sigma \to L$ be a countably additive vector measure. If f is a measurable function, it is said that it is scalarly integrable if it is integrable with respect to each scalar measure like $\lambda_{x'}$ for every $x' \in L'$, where $\lambda_{x'}(A) := \langle \lambda(A), x' \rangle$, $A \in \Sigma$. If f is scalarly integrable, it is said that it is integrable with respect to λ (λ-integrable for short) if for every $A \in \Sigma$ there is a vector $\int_A f\, d\lambda \in L$ such that $\langle \int_A f\, d\lambda, x' \rangle = \int_A f\, d\lambda_{x'}$. The definition of integrability of scalar functions with respect to a vector measure was first given by Bartle, Dunford and Schwartz [**1**] and studied by Lewis [**11**] and [**12**].

The Banach lattice $L_1(\lambda)$, see for example [**4**], is defined by the equivalence classes of λ-integrable functions such that the set where they differ has zero semi-variation, with the natural order and the norm

$$\|f\|_\lambda = \sup\Big\{ \int_\Omega |f|\, d|\langle \lambda, x' \rangle| : x' \in B_{X'} \Big\}, \quad f \in L_1(\lambda),$$

where $|\langle \lambda, x' \rangle|$ denotes the variation of the scalar measure $\lambda_{x'}$. The following expression gives an equivalent norm,

$$|||f|||_\lambda = \sup_{A \in \Sigma} \| \int_A f\, d\lambda \|, \quad f \in L_1(\lambda),$$

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