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A QUASILINEARIZATION APPROACH FOR TWO POINT NONLINEAR BOUNDARY VALUE PROBLEMS ON TIME SCALES

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ABSTRACT. In this paper the quasilinearization method is used in an approach to the unique solution of the separated boundary value problem on time scales from below and above by monotone convergent sequences of upper and lower solutions. The rate of convergence is also determined.

1. Introduction. In this paper we consider the separated boundary value problems (SBVPs)

$$-(p(t)x^{\Delta})^{\Delta} + q(t)x^{\sigma} = f(t, x^{\sigma}) + g(t, x^{\sigma}), \quad t \in [a, b]^{\kappa^2} \tag{1}$$

$$x(a) = A, \quad x(b) = B \tag{2}$$

and

$$-(p(t)x^{\Delta})^{\nabla} + q(t)x = f(t, x) + g(t, x), \quad t \in [a, b] \tag{3}$$

$$x(\rho(a)) = A, \quad x(\sigma(b)) = B. \tag{4}$$

In Section 2 we give some preliminary results with respect to the calculus on time scales which can also be found in the books by Bohner and Peterson [**7**] and Kaymakçalan, Lakshmikantham, and Sivasundaram [**12**]. In Section 3 we introduce the theory of the method of lower and upper solutions for the SBVP (1)–(2). Under certain assumptions on f and g we prove existence theorems for solutions of the SBVP (1)–(2) on a time scale $\mathbf{T}$. Then, in Section 4, the idea of the quasilinearization method is used for the SBVP (1)–(2) on $\mathbf{T}$ for which f and g are k-hyperconvex and k-hyperconcave functions, respectively. This method has been studied by Cabada and Nieto [**8**], Lakshmikantham and Vatsala [**13**], Mohapatra, Vajravelu and Yin [**14**]

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