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DEGENERATE ELLIPTIC SYSTEMS

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ABSTRACT. We solve the Riemann-Hilbert boundary value problem for a linearly elliptic system of two second order differential equations in a simply connected domain in the plane, which is degenerate on the whole boundary of the domain and reduced to a simple (canonical) form, whose characteristic equation has simple roots (to within low order terms).

1. Introduction. Degenerate elliptic equations and systems have extensive applications in mechanics. Such systems play a significant role in the theory of small bending surfaces and the membrane theory of shells with variable curvature [**16**].

The degenerate equations also occur in the study of magnetohydrodynamic streams, when the velocity exceeds the velocity of sound, as well as in the study of the motion of water in an open channel, when the stream velocity is greater than the velocity of the spreading surface waves [**8, 14**].

Such elliptic systems are reasonably well understood. It is known that the Dirichlet problem is Fredholm for one elliptic equation (this means that the homogeneous problem and the corresponding conjugate problem have the same finite number of linearly independent solutions), but this is not true for elliptic *systems*. For example, the Dirichlet problem for Bitsadze's system

$$u_{xx} - u_{yy} - 2v_{yy} = 0, \quad 2u_{xy} + v_{xx} - v_{yy} = 0$$

is neither Fredholm nor Noetherian (the given problem is called Noetherian if the reciprocal conjugate homogeneous problem has a finite number of linearly independent solutions) [**7**].

The main question in this context is: For what kind of boundary conditions will the problem be Noetherian for the degenerate elliptic system?

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