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HELLY'S SELECTION PRINCIPLE FOR FUNCTIONS OF BOUNDED P-VARIATION

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ABSTRACT. The classical Helly's selection principle states that a uniformly bounded sequence of functions with uniform bounded variation admits a subsequence which converges pointwise to a function of bounded variation. Helly's selection principle for metric space-valued functions of bounded p-variation is proven answering a question of Chistyakov and Galkin.

1. Introduction. Jordan introduced the concept of *variation* of a function and characterized functions of bounded variation as differences of nondecreasing functions. Helly [**10**, p. 222] used this decomposition to prove a compactness theorem for functions of bounded variation which has become known as Helly's selection principle, a uniformly bounded sequence of functions with uniform bounded variation has a pointwise convergent subsequence.

The interest in Helly's selection principle is natural since it provides an effective means of proving existence theorems in analysis. For some examples see [**3**] and [**9**]. A problem of importance is proving Helly type selection theorems for functions of generalized variation. For example, Helly's Selection Principle has been proven by Fleischer and Porter [**7**] for metric-space valued BV functions, Waterman [**11**] for functions of bounded Λ-variation, and Cyphert and Kelingos [**4**] for functions of bounded χ-variation.

The p-variation, $p \geq 1$, may be defined for a metric space-valued function $f : E \to X$ of a real variable as

$$V_p(f, E) = \sup \sum_{i=1}^{m} d(f(t_i), f(t_{i-1}))^p$$

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