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PHASE FUNCTIONS AND CENTRAL DISPERSIONS IN THE THEORY OF LINEAR FUNCTIONAL EQUATIONS

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ABSTRACT. The terms introduced in this paper, like phase function, conjugate numbers induced by a phase function, fundamental numbers and fundamental orbits, fundamental central dispersion of the phase function and central dispersions of higher orders, were studied by Borůvka and Neuman in connection with properties of solutions of linear differential equations.

A new direction is taken in this paper in order to remove the explicit dependence of the ideas upon differential equations. The theory presented here begins by defining anew the terms aforementioned, based only on properties of continuous functions, rather than by means of solutions to differential equations. The central idea for the generalized definitions is in a cyclic group of continuous functions, which effectively replaces the differential equation, giving a new direction to the original ideas of Borůvka and Neuman. The direction is similar to, but different than, the *unrestricted n-parameter family* theory introduced by Hartman, which generalizes solution properties of nth order linear differential equations to an abstract setting devoid of differential equations.

Some applications are given for using phase function ideas to solve certain linear functional equations of higher order and special linear difference equations with constant coefficients. These examples do not have an underlying differential equation and therefore the Borůvka-Neuman theory does not apply.

1. Introduction. The theories of phase functions and central dispersions have been treated by Borůvka [**1**] for solution spaces of second-order homogeneous linear differential equations in the Jacobi form. In [**7**], Neuman considered the same theories for nth order linear differential equations. In [**3**], Hartman considered conjugate point theory for nth order linear differential equations and generalized

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