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CONJUGACY SEPARABILITY OF CERTAIN HNN EXTENSIONS OF GROUPS

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ABSTRACT. In this paper we improve a criterion for the conjugacy separability of HNN extensions of conjugacy separable groups with cyclic associated subgroups. Using this result, we characterize the conjugacy separability of such HNN extensions of finitely generated nilpotent groups.

1. Introduction. A group G is said to be *conjugacy separable* if, for each pair $x, y \in G$ such that x and y are not conjugate in G, there exists a finite homomorphic image $\overline{G}$ of G such that the images of x and y in $\overline{G}$ are not conjugate in $\overline{G}$. Conjugacy separability is related to the conjugacy problem for groups as observed by Mal'cev [**11**] and Mostowski [**13**]. In this paper we consider the conjugacy separability of HNN extensions of a conjugacy separable group A with cyclic associated subgroups $\langle h \rangle$ and $\langle k \rangle$:

$$G = \langle A, t : t^{-1}ht = k \rangle.$$

In general, residual and separability properties of HNN extensions depend very much on the choice of the associated subgroups. Meskin [**12**] showed that the Baumslag-Solitar group, $\langle a, t : t^{-1}a^{\alpha}t = a^{\beta} \rangle$, is residually finite if and only if $|\alpha| = 1$ or $|\beta| = 1$ or $\alpha \pm \beta = 0$. In this paper we show that this is also true for conjugacy separability. Baumslag and Tretkoff [**3**] showed that HNN extensions of residually finite groups with finite associated subgroups are residually finite. Collins [**4**] proved a similar result for HNN extensions of conjugacy separable groups. We will make use of these results in our paper. Shirvani [**18**], Andreadakis, Raptis and Varsos [**1, 17**], considered residual finiteness

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