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A SYSTEM OF PELLIAN EQUATIONS AND RELATED TWO-PARAMETRIC FAMILY OF QUARTIC THUE EQUATIONS

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ABSTRACT. We show that solving of the two-parametric family of quartic Thue equations

$$x^4 - 2mnx^3y + 2\left(m^2 - n^2 + 1\right)x^2y^2 + 2mnxy^3 + y^4 = 1,$$

using the method of Tzanakis, reduces to solving the system of Pellian equations

$$V^2 - \left(m^2 + 2\right)U^2 = -2, \qquad Z^2 - \left(n^2 - 2\right)U^2 = 2,$$

where parameters m and $n \neq 0, \pm 1$ are integers. The main result in this paper can be stated as follows: If $|m|$ and $|n|$ are sufficiently large and have sufficiently large common divisor, then the system has only the trivial solutions $(V, Z, U) = (\pm m, \pm n, \pm 1)$, which implies that the original Thue equation also has only the trivial solutions $(x, y) = (\pm 1, 0), (0, \pm 1)$.

1. Introduction. Let $F \in \mathbf{Z}[X, Y]$ be a homogeneous irreducible polynomial of degree ≥ 3 and $t \neq 0$ a fixed integer. Then Diophantine equation $F(x, y) = t$ is called a Thue equation in honor of A. Thue, who proved in 1909 [**24**] that such an equation has only finitely many solutions $(x, y) \in \mathbf{Z} \times \mathbf{Z}$. Thue's proof is not effective. Using estimates for linear forms in logarithms of algebraic numbers, Baker [**1**] could give an effective upper bound for the solutions of Thue equation. Since that time, general powerful methods have been developed for the explicit solution of Thue equations, see [**21, 27, 5**], following from Baker's work. In 1990, Thomas [**23**] investigated for the first time a parametrized family of Thue equations. Since then, several families have been studied, see [**12**] for references. In particular, quartic families have been considered in [**6, 10, 12, 14, 16, 20, 22, 25, 28, 29**].

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