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ANALYTIC FUNCTIONS WITH H^p-DERIVATIVE

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ABSTRACT. For f an analytic function in the unit disc Δ, the following results are well known:

(1) If $f' \in H^p$ with $0 < p < 1$ then $f \in H^{p/(1-p)}$.

(2) If $f' \in H^1$, then f belongs to the disc algebra.

(3) If $f' \in H^p$ with $1 < p < \infty$, then f belongs to the Lipschitz space Λ_α with $\alpha = (p-1)/p$.

Both (1) and (2) have been shown to be sharp in a strong sense. We prove constructively that (3) is also very strongly sharp.

In spite of what we have just said, Aleman and Cima have recently obtained an improvement of (1) showing that a certain condition which is weaker than the condition $f' \in H^p$, $0 < p < 1$, is enough to conclude that $f \in H^{p/(1-p)}$. In this paper we also obtain the analogues of this for $p \geq 1$.

1. Introduction and main results. Let Δ denote the unit disc $\{z \in \mathbf{C} : |z| < 1\}$ and $\mathbf{T}$ the unit circle $\{\xi \in \mathbf{C} : |\xi| = 1\}$. For $0 \leq r < 1$ and f analytic in Δ, we set

$$M_p(r,f) = \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(re^{i\theta})|^p \, d\theta\right)^{1/p}, \quad \text{if } 0 < p < \infty,$$
$$M_\infty(r,f) = \max_{|z|=r} |f(z)|.$$

For $0 < p \leq \infty$ the Hardy space H^p consists of those functions f, analytic in Δ, for which

$$\|f\|_{H^p} \overset{\text{def}}{=} \sup_{0 \leq r < 1} M_p(r,f) < \infty.$$

There are a good number of classical and well-known results showing that the condition $f' \in H^p$ for a certain value of p implies that the function f belongs to a certain space of analytic functions.

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