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SUMMABILITY OF SPLICED SEQUENCES

JEFFREY A. OSIKIEWICZ

ABSTRACT. A spliced sequence is formed by combining all of the terms of two or more convergent sequences, in their original order, into a new "spliced" sequence. We investigate which nonnegative regular matrices will sum spliced sequences and to what value, and provide examples illustrating these results.

1. Preliminaries. Let c denote the set of convergent sequences and c_0 the set of null sequences. We write $e := \{1, 1, 1, \dots\}$ for the sequence all of whose terms are 1. If $x := \{x_n\}_{n=1}^{\infty}$ is a complex number sequence and $A := (a_{n,k})$ is a summability matrix, then Ax is the sequence whose nth term is given by $(Ax)_n := \sum_{k=1}^{\infty} a_{n,k} x_k$. The matrix A preserves zero limits if $x \in c_0$ implies $Ax \in c_0$, is regular if $\lim_n x_n = L$ implies $\lim_n (Ax)_n = L$, and is t-multiplicative, $t \in \mathbf{R}$, if $\lim_n x = L$ implies $\lim_n (Ax)_n = tL$. The well-known Silverman-Töeplitz theorem characterizes regular matrices, see [**1**].

Theorem 1.1 (Silverman, Töeplitz). *The matrix $A := (a_{n,k})$ is regular if and only if it satisfies the following three conditions*:

($\mathbf{Sp}_0$) $\lim_n a_{n,k} = 0$ *for each* $k = 1, 2, 3, \dots$;

($\mathbf{Zs}_1$) $\lim_n \sum_{k=1}^{\infty} a_{n,k} = 1$;

(Zn) $\sup_n \sum_{k=1}^{\infty} |a_{n,k}| < \infty$.

It can be shown, see [**1**], that A preserves zero limits if and only if it satisfies conditions (Sp_0) and (Zn), and is t-multiplicative if and only if it preserves zero limits and satisfies

(Zs) $\lim_n \sum_{k=1}^{\infty} a_{n,k} = t$.

Let A be a nonnegative regular matrix and E a subset of $\mathbf{N}$. Following Freedman and Sember [**3**], we define the A-density of E, denoted $\delta_A(E)$,

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