ABSTRACT. Let $\Phi$ be an analytic self-map of the disc, and let $H^p$ denote the Hardy space. The operator $DC_\Phi$ is defined for functions analytic in the disc by $DC_\Phi(f) = (f \circ \Phi)'$. We show that compactness and boundedness of the map $DC_\Phi : H^p \to H^q$, $p, q \geq 1$, are equivalent to the conditions $\Phi' \in H^q$ and $\|\Phi\|_\infty < 1$. For $\alpha > -1$ and $p \geq 1$, $A_\alpha^p$ denotes the weighted Bergman space. In the case $1 \leq p \leq q$, $DC_\Phi : A_\alpha^p \to A_\beta^q$ is bounded if and only if a related measure obeys a Carleson-type condition. Compactness is characterized by the analogous little-oh condition. For $1 \leq q < p$, Khinchine's inequality is used to show that boundedness and compactness are equivalent to an integrability condition on a weighted integral.

1. The Hardy space $H^p$, $p \geq 1$, is the Banach space of functions analytic in $U = \{z : |z| < 1\}$ satisfying

$$
\|f\|_{H^p} = \sup_{0 < r < 1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta \right\}^{1/p} < \infty.
$$

References for the Hardy spaces include [2] and [3].

Let $\Phi$ be a nonconstant self-map of $U$, and let $C_\Phi(f) = f \circ \Phi$ for functions $f$ analytic in the disc. Many authors [1, 6, 7, 10] have studied boundedness and compactness of $C_\Phi$ on the Hardy spaces. It is known [12] that if $C_\Phi$ is compact on $H^p$ for some $p \geq 1$, then $C_\Phi$ is compact on all the Hardy spaces. Shapiro [11] characterized the self-maps $\Phi$ for which $C_\Phi$ is compact on $H^2$.