ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 4, 2005

q-TRIPLICATE INVERSE SERIES RELATIONS WITH APPLICATIONS TO q-SERIES

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ABSTRACT. q-triplicate inverse series relations are obtained and used to derive terminating summation formulas of q-series which include the generalization of Gessel and Stanton's result.

1. Introduction.

1.1 Notation and basic hypergeometric series. Here we recall some standard notation for q-series, and basic hypergeometric series [**6**].

Given a (fixed) complex number q with $|q| < 1$, the basic hypergeometric series is defined by

$$ {}_{r+1}\phi_r \left[\begin{matrix} a_1, a_2, \cdots, a_{r+1} \\ b_1, b_2, \cdots, b_r \end{matrix} ; q, z \right] = \sum_{n=0}^{\infty} \frac{(a_1;q)_n \cdots (a_{r+1};q)_n}{(q;q)_n (b_1;q)_n \cdots (b_r;q)_n} z^n, $$

where, as before, the q-shifted factorial $(a;q)_n$ is given by

$$ (a;q)_n := (1-a)(1-aq)\cdots(1-aq^{n-1}), \quad n \geq 1, \quad (a;q)_0 := 1. $$
$$ (a;q)_{-n} := [(1-aq^{-1})(1-aq^{-2})\cdots(1-aq^{-n})]^{-1}, \quad n \geq 1. $$

For brevity, we employ the usual notation

$$ (a_1, a_2, \ldots, a_m; q)_n = (a_1;q)_n (a_2;q)_n \cdots (a_m;q)_n, $$
$$ (a_1, a_2, \ldots, a_m; q)_\infty = (a_1;q)_\infty (a_2;q)_\infty \cdots (a_m;q)_\infty. $$

A basic hypergeometric series ${}_{r+1}\phi_r$ is called very well-poised if $a_i b_i = q a_0$ for $i = 1, 2, \ldots, r$, and among the parameters a_i occur

2000 AMS *Mathematics Subject Classification.* Primary 33D15, Secondary 15A09.
Key words and phrases. Basic hypergeometric series, q-triplicate inverse series relations.
Received by the editors on December 16, 2002, and in revised form on April 29, 2003.

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