ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 4, 2005

# A STUDY OF THE LIPPMANN-SCHWINGER EQUATION AND SPECTRA FOR SOME UNBOUNDED QUANTUM POTENTIALS

CHRISTOPHER WINFIELD

ABSTRACT. In this article we study the Modified Lippmann–Schwinger equation for certain model potentials $V$ defined on $\mathbf{R}^3$, not of Rollnik class, and solutions to the equation in a weak sense. Further, we study the resolvent and the spectrum of the operator $H = -\Delta + cV$ in our model for nonzero constants $c$. In particular, we find that, for sufficiently small $c > 0$, $H$ has no singular spectrum.

**Introduction.** This article involves the study of the integral operator

$$(A_\lambda \phi)(x) = \frac{1}{4\pi} \int_{\mathbf{R}^3} \frac{|V(x)|^{1/2} e^{i\lambda|x-y|} V(y)^{1/2}}{|x-y|} \phi(y,\kappa) d\, y, \tag{0.1}$$

for certain classes of real-valued functions $V$ defined on $\mathbf{R}^3$ where $A_\lambda$ operates on a Hilbert space of functions $\phi$ also defined on $\mathbf{R}^3$ and where $\lambda$ is a complex parameter. Here $V$ is regarded as the potential for a (three-dimensional) Schrödinger operator $H \stackrel{\text{def}}{=} H_o + V = -\Delta + V$. We study a norm by Friedrichs [**1**] to develop a class of potentials $V$ for which $A_\lambda$ is not a Hilbert-Schmidt operator for any real $\lambda$, yet is compact for all real $\lambda$.

We apply our study of the operators $A_\lambda$ to the so-called modified Lippmann-Schwinger equation:

$$\begin{aligned} \psi(x,\kappa) = {} & |V(x)|^{1/2} e^{i\kappa\cdot x} \\ & - \frac{1}{4\pi} \int_{\mathbf{R}^3} \frac{|V(x)|^{1/2} e^{i|\kappa||x-y|} V(y)^{1/2}}{|x-y|} \psi(y,\kappa) d\, y. \end{aligned} \tag{0.2}$$

2000 AMS *Mathematics Subject Classification.* Primary 45E10, Secondary 47A40, 81U05.
Received by the editors on April 8, 2002, and in revised form on August 20, 2003.