

SEMIUMBILICS AND 2-REGULAR IMMERSIONS OF SURFACES IN EUCLIDEAN SPACES

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ABSTRACT. The semiumbilics are points at which the curvature ellipse of a surface degenerates to a segment. We characterize them here as critical points of the principal configurations associated to essential normal fields on the surface. This allows us to show that orientable closed surfaces with nonvanishing Euler number must have semiumbilics when immersed in 4-space. We also obtain as a consequence some conclusions relating the existence of 2-regular embeddings of surfaces in \mathbf{R}^5 in the sense of Feldman with that of globally defined essential normal fields.

1. Introduction. Feldman introduced in [4] the concept of regular immersion of order 2 of a submanifold in \mathbf{R}^n . He showed that, in the case of a surface, the most relevant dimensions to study this property are $n = 5, 6$, for there are no regular immersions of order 2 for $n \leq 4$ and, on the other hand, such immersions become generic when $n > 6$. A natural question in this context is that of the existence of 2-regular immersions for a given surface in \mathbf{R}^n , $n = 5, 6$. The answer is nontrivial, it is not at all easy to obtain 2-regular immersions of, for instance, a 2-sphere in \mathbf{R}^5 . A well-known one is the Veronese surface [3], but you can see that a small perturbation of this introduces 2-singular points [9]. It was observed in [9] that for $n = 5$ this problem can be put in terms of the contacts of the surface with hyperplanes. We point out that in a first stage these contacts are governed by the behavior of the second fundamental form of the surface, which is also related to that of the principal configurations associated to normal fields on the surface. Our purpose here is to use these tools in order to obtain further information on the possibility of obtaining 2-regular immersions of surfaces.

We find in the way that one obstruction for the 2-regularity of the surface is the existence of semiumbilics. Such points are defined in

Work of the first author partially supported by CAPES grant no. BEX 1191/99-3. Work of the second author partially supported by DGICYT grant no. BFM2003-02037.

Received by the editors on September 11, 2002 .