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KRULL RINGS, PRÜFER v-MULTIPLICATION RINGS AND THE RING OF FINITE FRACTIONS

THOMAS G. LUCAS

ABSTRACT. This paper deals with extending the notions of Krull domains and PvMDs to rings with zero divisors. Two of the problems to be addressed involve characterizing when the Nagata ring $R(x)$ will be a Krull ring and when it will be a PvMR. For both problems, the characterizations require consideration of how the ring in question sits in its associated ring of finite fractions.

1. Introduction. Throughout this paper, R will denote a commutative ring with identity, $T(R)$ will denote the total quotient ring of R and $Z(R)$ will denote the set of zero divisors of R. We use $Q_0(R)$ to denote the ring of finite fractions over R. One way to view the ring $Q_0(R)$ is to consider it as the subring of $T(R[X])$ which consists of those fractions $f = b(X)/a(X)$ where $a(X), b(X) \in R[X]$ with $\deg(b(X)) \leq \deg(a(X))$ such that $fa_i = b_i$ for each coefficient a_i of $a(X)$. Another is to view it as a set of equivalence classes of R-module homomorphisms on semiregular ideals, i.e., on those ideals of R which contain a finitely generated ideal that has no nonzero annihilators. Each class consists of those homomorphisms which agree on some semiregular ideal. In the next section we provide a few details of both these constructions. Note that if R is a McCoy ring, i.e., each finitely generated ideal containing only zero divisors has a nonzero annihilator, then $Q_0(R) = T(R)$. As each polynomial ring is a McCoy ring, [**42**, Proposition 6] and [**19**, Theorem 1], $Q_0(R[X]) = T(R[X])$.

While the results in this paper will hold for integral domains, the emphasis is on rings which have nonzero divisors of zero. Recall that an element of a ring R is said to be regular if it is not a zero divisor and an ideal is regular if it contains a regular element. Although an element is either regular or a zero divisor, an ideal need not be regular to have no

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