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THE POINTWISE VIEW OF DETERMINACY: ARBOREAL FORCINGS, MEASURABILITY, AND WEAK MEASURABILITY

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ABSTRACT. We prove that for all standard arboreal forcing notions $\mathbf{P}$ there is a counterexample for the implication "If A is determined, then A is $\mathbf{P}$-measurable". Moreover, we investigate for which forcing notions this is extendible to "weakly $\mathbf{P}$-measurable".

1. Introduction. The use of coding techniques is ubiquitous in the theory of determinacy: we code countably many reals as one, finite sequences as natural numbers, and basic open sets as finite sequences. A consequence of this fact is that many proofs using determinacy do not work with the set under investigation but with some coded or decoded version. Since most of the literature on determinacy works with the assumption that a pointclass respected by the coding is determined, this is not a problem.

As soon as we do not talk about pointclasses anymore but move on to individual sets, we start getting into trouble:

For instance, if we ask whether a given determined set has nice properties, for example, the Baire property, we tend to get unpleasant answers: In general, determined sets can be as nasty as you want them to be, as determinacy is a very local property.

This paper is part of a project trying to understand the consequences of determinacy for individual sets better. The difference between pointwise and classwise views of determinacy plays a role in higher set theory, e.g., being homogeneously Suslin has pointwise consequences, while being determined usually has only classwise consequence, and the present author has used counterexamples like the ones constructed in this paper in [**12**] to show that the usual proof of Turing determinacy will not work under the assumption of imperfect information determinacy.

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