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# ON THE ZEROES OF TWO FAMILIES OF POLYNOMIALS ARISING FROM CERTAIN RATIONAL INTEGRALS

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ABSTRACT. We prove a conjecture of Boros, Moll and Shallit on the location of the zeroes of certain polynomials arising in the evaluation of the rational definite integrals $\int_0^\infty [dx/(x^4+2ax^2+1)^{m+1}]$.

**1. Introduction.** In a series of recent papers George Boros, Victor Moll, and a number of coauthors have studied patterns in closed form expressions for the rational definite integrals

$$\int_0^\infty \frac{dx}{(x^4+2ax^2+1)^{m+1}}, \quad a > -1. \tag{1}$$

The recent article [**4**] gives some of the interesting background behind this line of investigation and a survey of their results. In this note we will take up a question concerning certain polynomials connected to the integrals (1) introduced in [**1**].

A standard argument shows that (1) is equal to

$$\frac{\pi\binom{2m}{m}}{2^{3m+3/2}(a+1)^{m+1/2}}\ {}_2F_1(-m, m+1;\ 1/2-m;\ (1+a)/2),$$

where ${}_2F_1$ is the usual hypergeometric series. For positive integral $m$, the hypergeometric series terminates and the authors of [**1**] study the polynomials in the variable $a$ defined by $P_m(a) = \binom{2m}{m}\ {}_2F_1(-m, m+1; 1/2-m; (1+a)/2)$. For each $m$, $P_m(a)$ is a polynomial all of whose coefficients are positive integers.

Let $d_l(m)$ be the coefficient of $a^l$ in $P_m(a)$. In [**1**], it is shown that

$$d_l(m) = \frac{1}{l!m!2^{m+l}}\left(\alpha_l(m)\prod_{k=1}^{m}(4k-1) - \beta_l(m)\prod_{k=1}^{m}(4k+1)\right),$$

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