

## JACOBI AND MODULAR FORMS ON SYMMETRIC DOMAINS

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**ABSTRACT.** We prove that there is an isomorphism between the space of Jacobi forms on a symmetric domain associated to an equivariant holomorphic map and the space of a certain vector-valued modular form of half-integral weight on the given symmetric domain. We also construct Eisenstein series for Jacobi forms on symmetric domains and express those in terms of theta functions.

**1. Introduction.** Jacobi forms on the Poincaré upper half plane or more generally on Siegel upper half spaces generalize classical theta functions, and they arise naturally as coefficients of Siegel modular forms, cf., [3, 17]. As is expected, they play an important role in number theory, and various arithmetic aspects of such Jacobi forms have been investigated in numerous papers, see, e.g., [5, 7, 11, 14]. On the geometric side, they are closely linked to elliptic surfaces over modular curves or families of abelian varieties parametrized by Siegel modular varieties, see [8, 15]. Jacobi forms are also related to elliptic genera of complex manifolds, and they appear as partition functions of super symmetric sigma models whose target spaces are Calabi-Yau manifolds, see [4].

The Poincaré upper half plane and Siegel upper half spaces are special cases of (Hermitian) symmetric domains, and Jacobi forms can also be defined on symmetric domains. For example, Borcherds studied Jacobi forms on symmetric domains associated to orthogonal groups, cf., [1, 2]. We are interested in symmetric domains which allow equivariant holomorphic maps into Siegel upper half spaces. Let  $\mathcal{D}$  be a symmetric domain associated to a semi-simple Lie group  $G$  of Hermitian type, and let  $\mathcal{H}_n$  be the Siegel upper half space of degree  $n$ . We assume that there is a holomorphic map  $\tau : \mathcal{D} \rightarrow \mathcal{H}_n$  that is equivariant with respect to a homomorphism  $\rho : G \rightarrow \mathrm{Sp}(n)$  of Lie groups. Then we can construct

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