ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 4, 2005

INFINITELY MANY RADIAL AND NON-RADIAL SOLUTIONS FOR A CLASS OF HEMIVARIATIONAL INEQUALITIES

ALEXANDRU KRISTÁLY

ABSTRACT. This paper is concerned with the existence of infinitely many radial respective non-radial solutions for a class of hemivariational inequalities, applying the non-smooth version of the fountain theorem. The main tool used in our framework is the principle of symmetric criticality for a locally Lipschitz functional which is invariant under a group action.

1. Introduction. Let $F : \mathbf{R}^N \times \mathbf{R} \to \mathbf{R}$ be a Carathéodory function which is locally Lipschitz in the second variable, fulfilling the following condition:

(F_1) there exist $c_1 > 0$ and $p \in]2, 2^*[$ such that

$$|\xi| \leq c_1(|s| + |s|^{p-1}), \quad \forall\, \xi \in \partial F(x,s), \quad \text{for a.e. } x \in \mathbf{R}^N, \quad \forall\, s \in \mathbf{R},$$

where $N \geq 2$ and $p \in]2, 2^*[$, $2^* = 2N/(N-2)$, if $N \geq 3$ and $2^* = \infty$, if $N = 2$, and $F(x,0) = 0$ almost everywhere $x \in \mathbf{R}^N$.

The set

$$\partial F(x,s) = \{\xi \in \mathbf{R} : \xi z \leq F_x^0(x,s;z) \text{ for all } z \in \mathbf{R}\}$$

is the generalized gradient of $F(x,\cdot)$ at $s \in \mathbf{R}$, where

$$F_x^0(x,s;z) = \limsup_{\substack{y \to s \\ t \to 0^+}} \frac{F(x,y+tz) - F(x,y)}{t},$$

is the generalized directional derivative of $F(x,\cdot)$ at the point $s \in \mathbf{R}$ in the direction $z \in \mathbf{R}$, see Clarke [**9**].

Key words and phrases. Hemivariational inequalities, principle of symmetric criticality, locally Lipschitz functions, Palais-Smale condition, radial and non-radial solutions.
Received by the editors on November 22, 2002, and in revised form on April 20, 2003.