

LINKS AMONG CHARACTERISTICALLY NILPOTENT, C -GRADED AND DERIVED FILIFORM LIE ALGEBRAS

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ABSTRACT. The aim of this paper is to study the interconnectivity among Lie algebras that are characteristically nilpotent, derived and c -graded filiform, establishing some characterization theorems involving them.

At the same time we offer an algorithm that will allow us to know, given a complex filiform Lie algebra, which of those properties hold.

Introduction. Although there is a lot of research on Lie algebras, it often studies those that are characteristically nilpotent, derived or c -graded filiform in isolation and characterization theorems do not usually consider possible connections among these kinds of Lie algebras. In an attempt to fill this gap this paper tries to give some characterization theorems of characteristically nilpotent Lie algebras involving derived Lie algebras or c -graded filiform Lie algebras. Indeed, the main results obtained are Theorems 3.2 and 4.1.

We also show an algorithmic procedure to check if, given a complex filiform Lie algebra, it is, under certain conditions, characteristically nilpotent, a derived Lie algebra or a c -graded filiform Lie algebra.

The structure of this paper is the following. In Section 1 we briefly show the historical evolution of these three kinds of Lie algebras, since they were first introduced. These historical notes could explain how the main difficulties in the study of these Lie algebras appeared and how they were solved. Section 2 recalls the definitions and main properties that will be used in the following sections of the paper. Section 3, on the other hand, is devoted to the study of the connection between characteristically nilpotent filiform Lie algebras and derived Lie algebras. First we give two nilpotent Lie algebras (those of the lowest possible dimension) which are neither derived nor

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