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GERMS OF HOLOMORPHIC FUNCTIONS ON TOPOLOGICAL VECTOR SPACES AND INVARIANT RINGS

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ABSTRACT. Let V be a locally convex and Hausdorff topological vector space and G a finite group of holomorphic automorphisms of $O_{V,0}$. Here we prove that the ring $O_{V,0}^G$ of all invariant germs is a C.M.$_\infty$-local ring.

1. Introduction. Let (X, O_X) be a Hausdorff reduced complex space locally embedded in a locally convex topological vector space, i.e., a Cartan space with the terminology of [**2**, p. 65] and G a finite group of holomorphic automorphisms of X. Let X/G be the set of all G-orbits equipped with the quotient topology, and let $f : X \to X/G$ be the quotient map. Since G is finite, X/G is Hausdorff. For any open subset Ω of X/G, let $H^0(\Omega, O_\Omega) := H^0(\pi^{-1}(\Omega), O_{\pi^{-1}(\Omega)})^G$ be the set of all G-invariant holomorphic functions on $\pi^{-1}(\Omega)$. In this way we obtain a sheaf $O_{X/G}$ of local $\mathbf{C}$-algebras on X/G. In general the local rings $O_{X/G,P}$ are not Noetherian. Here we study the Cohen-Macaulyness of the local rings of X/G, X smooth, in the non-Noetherian case. For a theory of grade in the non-Noetherian case, see [**1**] or [**2**, Chapter 1]. We recall here the definition of C.M.$_\infty$-local ring given in [**2**, pp. 34–35]. Let A be a unitary commutative ring and n a nonnegative integer. For any A-module M, let $T_n(M)$ denote the set of all $x \in M$ such that the annihilator of x has grade at least n. It is easy to see that $T_n(M/T_n(M)) = 0$ and $T_nT_n = T_n$. Thus the functor T_n defines a torsion theory, i.e., for all submodules N of M we may define the n-closure of N in M as the inverse image in M of $T_n(M/N)$. The ring A, respectively the module M, is said to be n-Noetherian if each increasing sequence of n-closed ideals of A, respectively n-closed submodules of M, is stationary and ∞-Noetherian if it is n-Noetherian for all n. The

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