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COHEN-MACAULAY DIMENSION OF MODULES OVER NOETHERIAN RINGS

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ABSTRACT. We extend a criterion of Gerko for a ring to be Cohen-Macaulay to arbitrary, not necessarily local, Noetherian rings. Our version reads as follows: The Noetherian ring R is Cohen-Macaulay if and only if, for all finitely generated R-modules M, CM-$\dim_R M$ is finite.

1. Introduction. There are many important homological dimensions, defined for finitely generated module M over a commutative Noetherian ring R. The classic one is projective dimension P-dim, which characterizes regular rings by a famous result of Auslander, Buchbaum and Serre. Another dimension corresponding to the complete intersection property of ring is defined by Avramov, Gasharov and Peeva [**4**] and is denoted by CI-dim. Gerko also defined a dimension which reflects the complete intersection property of the ring called polynomial complete intersection dimension and denoted PCI-dim [**7**]. Oana Veliche [**9**] called it lower complete intersection dimension and used notion CI_*-dim to denote it. The notion of G-dimension was introduced by Auslander and Bridge, denoted G-dim, and has some relation to the Gorenstein property of R [**1**]. There is another dimension, defined by Veliche, called upper Gorenstein dimension or G^*-dimension, denoted G^*-dim that characterizes Gorenstein local rings. Dimension which reflects Cohen-Macaulay property of rings is defined also by Gerko, called Cohen-Macaulay dimension and denoted CM-dim [**7**].

Putting them together and using the same terminology as in [**9**], we have notions of homological dimensions of finitely generated module M, denoted H-$\dim_R M$ for H=P, CI, CI_*, G, G^* or CM. We say that, not necessary local, ring R has property (H) with H=P, (respectively,

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