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CLASSIFICATION OF 3-DIMENSIONAL ISOLATED RATIONAL HYPERSURFACE SINGULARITIES WITH $\mathbf{C}^*$-ACTION

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1. Introduction. In [**2**] Artin first introduced the definition of rational surface singularity. He classified all rational surface singularities embeddable in $\mathbf{C}^3$. These are precisely those Du Val singularities in $\mathbf{C}^3$ defined by one of the following polynomial equations:

$$\begin{aligned} A_n:&\quad x^2+y^2+z^{n+1}, &&\text{for } n\geq 1\\ D_n:&\quad x^2+y^2z+z^{n-1}, &&\text{for } n\geq 4\\ E_6:&\quad x^2+y^3+z^4\\ E_7:&\quad x^2+y^3+yz^3\\ E_8:&\quad x^2+y^3+z^5. \end{aligned}$$

It is well known that any canonical singularity, i.e., singularity that occurs in a canonical model of a surface of general type, is analytically isomorphic to one of the rational double points listed above.

In [**3**] Burns defined higher dimensional rational singularity as follows. Let (V,p) be an n-dimensional isolated singularity. Let $\pi: M\to V$ be a resolution of singularity. And p is said to be a rational singularity if $R^i\pi_*\mathcal{O}_M=0$ for $1\leq i\leq n-1$. In [**14**], Yau shows for Gorenstein singularity that it is sufficient to require $R^{n-1}\pi_*\mathcal{O}_M=0$. He [**14**] proves that $R^{n-1}\pi_*\mathcal{O}_M\cong H^0(V-\{p\},\Omega^n)/L^2(V-\{p\},\Omega^n)$ where Ω^n is the sheaf of germs of holomorphic n-forms and $L^2(V-\{p\},\Omega^n)$ is the space of holomorphic n forms on $V-\{p\}$ which are L^2-integrable. The geometric genus p_g of the singularity (V,p) is defined to be

$$p_g:=\dim R^{n-1}\pi_*\mathcal{O}_M=\dim H^0(V-\{p\},\Omega^n)/L^2(V-\{p\},\Omega^n).$$

It turns out that p_g is an important invariant of (V,p).

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