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SYMPLECTIC GEOMETRY FOR PAIRS OF SUBMANIFOLDS

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ABSTRACT. Darboux's classical theorem in symplectic geometry is generalized to pairs of transversal submanifolds.

1. Introduction. A smooth manifold V imbued with a closed, non-degenerate 2-form ω is called a symplectic manifold. The symplectic form ω gives the manifold a geometric structure (signed area, instead of length as in Riemannian geometry), and the closedness controls the topology of V. Symplectic manifolds play an important role in classical mechanics, geometrical optics, representation theory, and Kähler geometry. A variety of fundamental results in symplectic geometry provide for local characterizations of various geometric objects: symplectic manifolds, submanifolds, foliations, etc., the most fundamental and elementary of which is Darboux's theorem:

Theorem 1 (Darboux's theorem). *Every point of a symplectic manifold has local coordinates* (x_i, y_i), $i = 1, \dots, n$, *so that*

$$\omega = dx_1 \wedge dy_1 + \cdots + dx_n \wedge dy_n.$$

We can conclude that, in stark contrast to Riemannian geometry, there are no local invariants other than dimension and that this dimension must be even. Another perspective on Darboux's theorem is this: Any two symplectic forms induce the same form on a point (the zero form) and so the intrinsic symplectic geometry of a point completely determines the symplectic geometry nearby.

In this paper we examine the extent to which the interior geometry of a pair of submanifolds determines its exterior geometry, a special

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