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GEOMETRY OF JUMP SYSTEMS

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ABSTRACT. A jump system is a set of lattice points satisfying a certain "two-step" axiom. We present a variety of results concerning the geometry of these objects, including a characterization of two-dimensional jump systems, necessary (though not sufficient) properties of higher-dimensional jump systems, and a characterization of constant-sum jump systems.

1. Introduction. A jump system is a set of lattice points that satisfy a simple "two-step" axiom. They were introduced by Bouchet and Cunningham [**1**] in order to simultaneously generalize delta-matroids (hence matroids) and degree sequences of subgraphs.

Fix a finite set S. We consider elements of $\mathbf{Z}^S$ together with the 1-norm $|x| = \sum_{i\in S} |x_i|$ and the corresponding distance $d(x,y) = |x-y|$.

For elements $x, y \in \mathbf{Z}^S$, we say $z \in \mathbf{Z}^S$ is a *step from* x *toward* (*in the direction of*) y if $|z-x| = 1$ and $|z-y| < |x-y|$. Note that if z is a step from x toward y, then $z = x \pm e_i$ for some standard unit vector e_i. For notational convenience, we will use $x \xrightarrow{y} z$ to denote a step from x to z in the direction of y.

Given a collection of points $J \subseteq \mathbf{Z}^S$, we say that J is a *jump system* if it satisfies Axiom 1.1.

Axiom 1.1 (2-step axiom). *If* $x, y \in J$ *and* $x \xrightarrow{y} z$ *with* $z \notin J$*, then there exists* $z' \in J$ *with* $z \xrightarrow{y} z'$.

The following well-known operations all preserve Axiom 1.1, see [**1, 3, 4, 5**]. They allow us to simplify many of the later proofs concerning various properties of jump systems.

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