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EXISTENCE AND PROPERTIES OF MULTIPLE POSITIVE SOLUTIONS FOR SEMI-LINEAR EQUATIONS WITH CRITICAL EXPONENTS

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1. Introduction and main results. In this paper we consider the following semi-linear elliptic problem

$$-\Delta u + u = u^p + \mu f(u+\phi) \tag{1.1}_\mu$$

$$u \in H^1(R^N), \quad u > 0 \quad \text{in } R^N \tag{1.2}$$

where $\mu \geq 0$ is a given constant, $p = (N+2)/(N-2)$ is the critical Sobolev's exponent. $\phi(x)$ is some given function in $L^1(R^N) \cap C^\alpha(R^N)$ and

H_1) $\phi(x) \geq 0$, $\phi(x) \not\equiv 0$ in R^N, $|x|^{N-2}\phi(x)$ is bounded.

The hypotheses for f(t) are as follows:

f_1) $f \in C^2(R^+)$, $\quad f'(t) \geq 0$, $f''(t) \geq 0$ for all $t \geq 0$.

f_2) There exists a $\delta > 0$ such that $tf'(t) \geq (1+\delta)f(t)$ for $t \geq 0$ if $N \geq 6$.

f_3) $\lim_{t\to 0} f(t)/t = 0$, and $\lim_{t\to\infty} f(t)/t^q = 0$ for some $q \geq p$.

f_4) $\lim_{t\to\infty} f(t)/t = +\infty$.

Critical semi-linear elliptic equations arise from widely diverse problems in differential geometry, quantum physics, astrophysics, and other scientific areas. Many researchers have studied the second order semi-linear elliptic boundary value problems involving critical exponents. Here we mention the articles written by Brezis and Nirenberg [**4**], Cerami, Fortunato and Struwe [**5**], Lions [**14**], Ambrosetti and Struwe [**2**]. In their papers, many interesting results about the existence and nonexistence have been obtained by using variational methods when nonlinear function is homogeneous. For the inhomogeneous case, Zhu

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