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PARAMETRIC SOLUTIONS OF THE QUARTIC DIOPHANTINE EQUATION $f(x,y) = f(u,v)$

AJAI CHOUDHRY

ABSTRACT. There are very few quartic diophantine equations of the type $f(x,y) = f(u,v)$, where $f(x,y) = ax^4 + bx^3y + cx^2y^2 + dxy^3 + ey^4$ is a binary quartic form, for which parametric solutions have been obtained. In this paper we obtain parametric solutions of such quartic equations when the coefficients a, b, c, d, e satisfy certain conditions.

1. Introduction. This paper is concerned with quartic diophantine equations of the type

$$f(x,y) = f(u,v) \tag{1.1}$$

where

$$f(x,y) = ax^4 + bx^3y + cx^2y^2 + dxy^3 + ey^4 \tag{1.2}$$

is a binary quartic form in the variables x and y, and the coefficients a, b, c, d and e are integers. At present no method is known of determining all integer solutions or even a single non-trivial solution of a given quartic equation of type (1.1). A necessary and sufficient condition for the solvability of equation (1.1) has been given by Choudhry [**3**]. Even when this condition is satisfied we may not get a parametric solution of the given equation. The only two equations of type (1.1) for which parametric solutions have been explicitly obtained are the classical equation

$$x^4 + y^4 = u^4 + v^4$$

for which several solutions are known [**1**], [**5**, p. 201], [**6**], and the equation

$$x^4 + 4y^4 = u^4 + 4v^4$$

for which a parametric solution has been given by Choudhry [**2**]. Further, Segre [**8**, pp. 388–390] has indicated a method of obtaining

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