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NON-EXISTENCE OF CERTAIN 3-STRUCTURES

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ABSTRACT. We introduce the notion of an ε-framed 3-structure. This is a general structure which includes many widely studied 3-structures (almost quaternion, almost contact, hyper f-structure, almost product, etc.). We prove the existence of Riemannian metrics compatible with such a structure. We also study particular cases of ε-framed 3-structures showing the non-existence of certain remarkable types of such structures. First, we prove the non-existence of P-Sasakian almost r-paracontact 3-structures. Then, we show the non-existence of almost r-contact $\mathcal{S}$-3-structures (with $r > 1$). Finally, we establish the non-existence of proper trans-Sasakian almost contact 3-structures. A consequence of this last result is that any b-Kenmotsu almost contact 3-structure must be hypercosymplectic.

1. Introduction. In 1963, Yano [**33**] introduced the notion of an f-structure on a manifold, which is defined by a non-null $(1,1)$ tensor field f satisfying $f^3 + f = 0$. The concept of an f-structure includes the notions of almost complex and almost contact structures and it is well known that it is genuinely a more general structure. For instance, hypersurfaces of almost contact manifolds are not in general almost complex manifolds, but they have always f-structures associated to them.

Almost product structures are another type of structure widely studied by several authors, see [**34, 21**]. Analogously to the situation for almost complex and almost contact structures, almost paracontact structures are closely related to almost product structures. The concept of an $f(3,\varepsilon)$-structure was introduced in [**30**] as a uniform way of treating all the above geometries and several others. An $f(3,\varepsilon)$-structure, $\varepsilon \in \{\pm 1\}$, is defined by a non-null $(1,1)$ tensor field f satisfying $f^3 - \varepsilon f = 0$. It turns out that f is of constant rank and there are two complementary distributions associated with the $f(3,\varepsilon)$-structure, as happens with f-structures and some other known cases.

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