ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 6, 2005

# CARATHÉODORY SOLUTIONS TO HYPERBOLIC FUNCTIONAL DIFFERENTIAL SYSTEMS WITH STATE DEPENDENT DELAYS

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ABSTRACT. The paper is concerned with initial problems for quasilinear systems of first order partial functional differential equations. The unknown function is the functional argument in equations, the partial derivatives appear in a classical sense. A theorem on the existence of a solution and continuous dependence upon initial data is proved. The Cauchy problem is transformed into a system of functional integral equations. The existence of a solution of this system is proved by using integral inequalities and the method of bicharacteristics.

Differential systems with deviated variables and differential integral systems can be derived from a general model by specializing given operators.

**1. Introduction.** For any metric spaces $U$ and $V$, let $C(U,V)$ denote the class of all continuous functions from $U$ into $V$. Let $L([0,c],R_+)$ where $c>0$ and $R_+=[0,+\infty)$ is the set of all functions $\eta:[0,a]\to R_+$ which are integrable on $[0,c]$. We will use vectorial inequalities with the understanding that the same inequalities hold between their corresponding components.

Denote by $M_{k\times n}$ the set of all matrices

$$X=[x_{ij}]_{i=1,\dots,k,\ j=1,\dots,n}$$

with real elements. For $x=(x_1,\dots,x_n)\in R^n$, $p=(p_1,\dots,p_k)\in R^k$ and $X\in M_{k\times n}$, we write

$$\|x\|=|x_1|+\cdots+|x_n|,\quad \|p\|=\max\{|p_i|:\ 1\le i\le k\},$$
$$\|X\|=\max\left\{\sum_{j=1}^{n}|x_{ij}|:\ 1\le i\le k\right\}.$$

2000 AMS *Mathematics Subject Classification.* Primary 35R10, 35F25.
Received by the editors on December 15, 2000, and in revised form on March 1, 2002.