INDUCED REPRESENTATIONS OF
LOCALLY C^*-ALGEBRAS

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ABSTRACT. In this paper, by analogy with the case of C^*-algebras, we define the notion of induced representation of a locally C^*-algebra and then we prove the imprimitivity theorem for induced representations of locally C^*-algebras.

1. Introduction. Locally C^*-algebras generalize the notion of C^*-algebra. A locally C^*-algebra is a complete Hausdorff complex topological $*$-algebra A whose topology is determined by its continuous C^*-seminorms in the sense that the net $\{a_i\}_{i \in I}$ converges to 0 if and only if the net $\{p(a_i)\}_{i \in I}$ converges to 0 for every continuous C^*-seminorm p on A. The terminology "locally C^*-algebra" is due to Inoue, see [**2**]. Locally C^*-algebras were also studied by Phillips, under the name of pro -C^*-algebra, see [**7**], Fragoulopoulou, and other people.

A representation of A on a Hilbert space H is a continuous $*$-morphism φ from A to $L(H)$, the C^*-algebra of all bounded linear operators on H. Given a locally C^*-algebra A which acts non-degenerately on a Hilbert module E over a locally C^*-algebra B and a non-degenerate representation (φ, H) of B, exactly as in the case of C^*-algebras, see [**8**], we construct a representation of A, called the Rieffel-induced representation from B to A via E, and then we prove some properties of this representation. Thus, we prove that the theorem on induction in stages, Theorem 5.9 in [**8**], is also true in the context of locally C^*-algebras, Theorem 3.6. In Section 4, we prove that if A and B are two locally C^*-algebras which are strong Morita equivalent, then any non-degenerate representation of A is induced from a non-degenerate representation of B, Theorem 4.4.

2. Preliminaries. Let A be a locally C^*-algebra, and let $S(A)$ be the set of all continuous C^*-seminorms on A. If $p \in S(A)$,