ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 35, Number 6, 2005

PERIODIC SOLUTIONS OF AN INFINITE DIMENSIONAL HAMILTONIAN SYSTEM

YANHENG DING AND CHENG LEE

ABSTRACT. We establish existence and multiplicity of periodic solutions to the infinite dimensional Hamiltonian system

$$\begin{cases} \partial_t u - \Delta_x u = H_v(t,x,u,v) \\ -\partial_t v - \Delta_x v = H_u(t,x,u,v) \end{cases} \quad \text{for } (t,x) \in \mathbf{R}\times\Omega,$$

where $\Omega\subset\mathbf{R}^N$ is a bounded domain or $\Omega = \mathbf{R}^N$. When Ω is bounded, we treat the situations where $H(t,x,z)$ is, with respect to $z=(u,v)$, sub- or superquadratic, or concave and convex, and discuss also the convergence to homoclinics of sequences of subharmonic orbits. If $\Omega = \mathbf{R}^N$, we handle the case of superquadratic nonlinearities.

1. Introduction. In this paper we are interested in existence and multiplicity of periodic orbits of the following system of partial differential equations

$$\text{(HS)} \qquad \begin{cases} \partial_t u - \Delta_x u = H_v(t,x,u,v) \\ -\partial_t v - \Delta_x v = H_u(t,x,u,v) \end{cases} \qquad \text{for } (t,x) \in \mathbf{R}\times\Omega.$$

Here $\Omega \subset \mathbf{R}^N$ is a smoothly bounded domain or $\Omega = \mathbf{R}^N$, $z=(u,v)$: $\mathbf{R}\times\Omega \to \mathbf{R}^m \times \mathbf{R}^m$, and $H \in \mathcal{C}^1(\mathbf{R}\times\bar{\Omega}\times\mathbf{R}^{2m},\mathbf{R})$, where $\overline{\Omega} = \Omega$ if $\Omega = \mathbf{R}^N$. Letting

$$\mathcal{J} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad \mathcal{J}_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

and $A = \mathcal{J}_0(-\Delta_x)$, (HS) can be rewritten as $\mathcal{J}(d/dt)z + Az = H_z(t,x,z)$. Certain linear and nonlinear problems connecting the operator $\mathcal{J}\partial_t - \mathcal{J}_0\Delta_x$ arise in optimal control of systems governed

2000 AMS *Mathematics Subject Classification.* Primary 35A15, 35K50.
Key words and phrases. Infinite-dimensional Hamiltonian system, periodic solutions, variational method.
Received by the editors on October 8, 2002, and in revised form on March 24, 2003.