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AN IMPLICIT FUNCTION THEOREM

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ABSTRACT. A nonsmooth variant of the implicit function theorem is proved.

Let $m, n \geq 1$ be integers. Denote by M_n the linear space of the $n \times n$ matrices with real elements, by I_n the unit matrix of M_n. Let $B^n(x, r)$ be the ball in $\mathbf{R}^n$ with center at the point x and radius $r > 0$.

If $F(x, y)$ is any locally Lipschitz vector-function of variables $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$ and (x, y) is a point of differentiability of F, then let $F'(x, y)$ be its Jacobi matrix, $F'_x(x, y)$ be the Jacobi matrix with respect to x for any fixed y and $F'_y(x, y)$ be the Jacobi matrix with respect to y for any fixed x.

For an arbitrary matrix $C \in M_n$ we put

$$|C| = \max_{|h|=1} |Ch|.$$

If $C(x) : D \subset \mathbf{R}^m \to M_n$ is a matrix function, then set

$$\|C\|_D = \operatorname{ess\,sup}_{x \in D} |C(x)|.$$

For $P \subset \mathbf{R}^m$ let $K : P \subset \mathbf{R}^m \to M_n$ be an arbitrary matrix function. We set

$$\operatorname{osc}(K, P) = \operatorname{ess\,sup}_{x,y \in P} |K(x) - K(y)|.$$

We shall prove the following nonsmooth variant of the well-known implicit function theorem.

Theorem. *Let $x_0 \in \mathbf{R}^n$, $y_0 \in \mathbf{R}^m$. Let $D = B^n(x_0, r') \times B^m(y_0, r'')$ be a domain and $F : D \to \mathbf{R}^m$ be a locally Lipschitz mapping. Suppose that*

$$\mu \equiv \|F'_y - I_m\|_D + \operatorname{osc}(F'_x, D)\,(1 + \|F'\|_D) < 1. \tag{1}$$

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