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ON THE CLASSIFICATION THEOREMS OF ALMOST-HERMITIAN OR HOMOGENEOUS KÄHLER STRUCTURES

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ABSTRACT. A proof by Young tableaux and symmetrizers is given of the classification theorems by Gray and Hervella of almost-Hermitian structures and by Abbena and Garbiero of homogeneous Kähler structures.

1. Introduction. As it is well known, representation theory has been applied to the classification of several geometric structures on differentiable manifolds, beginning with the almost-Hermitian structures [**10**].

An interesting case is that of homogeneous Kähler structures [**1, 4, 6**], both because of the importance of the manifolds under study and also as it gives some specific examples of representations of the unitary group $U(n)$. Moreover, Abbena-Garbiero's classification [**1**] has found an application [**8**] to spaces of negative constant holomorphic sectional curvature: The characterization of the complex hyperbolic space as the only connected simply-connected irreducible homogeneous Kähler manifold admitting a nonvanishing homogeneous Kähler structure in Abbena-Garbiero's class $\mathcal{K}_2 \oplus \mathcal{K}_4$, see [**1**] and Section 2 below. On the other hand, the almost-Hermitian case also has much interest, see [**5**] amongst many others.

The aim of the present paper is to give a proof of Gray-Hervella's [**10**] and Abbena-Garbiero's [**1**] theorems, by using Young tableaux and symmetrizers. Although other demonstrations have been given [**4**–**6**], we think that one more proof is in order due to the importance of both theorems and because the present proof can perhaps aid to a better understanding of the involved decompositions, and to solve some related questions: For instance, the expression of the tensors in the classes in the homogeneous quaternionic Kähler case, with relevant

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