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EXISTENCE AND BEHAVIOR OF SOLUTIONS OF THE RATIONAL EQUATION $x_{n+1} = (ax_{n-1} + bx_n)/(cx_{n-1} + dx_n)x_n$, $n = 0, 1, 2, \ldots$

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ABSTRACT. We investigate the existence and behavior of the solutions of the equation in the title, where a, b, c, and d are real numbers, and the initial conditions are real numbers.

1. Introduction and preliminaries. Consider the equation

$$x_{n+1} = \frac{ax_{n-1} + bx_n}{cx_{n-1} + dx_n}\, x_n, \quad n = 0, 1, \ldots \tag{1}$$

where the parameters

$$a, b, c, d$$

are given real numbers and the initial conditions x_{-1}, x_0 are arbitrary real numbers.

This work is motivated by Problem 1572 in *Mathematics Magazine*, April 1999, [**5**].

Our first goal is to give a detailed description of the set

$$\mathcal{G} = \{(x_{-1}, x_0) \in \mathbf{R}^2 : \text{Eq. (1) is well defined for all } n \geq 0\}.$$

The set $\mathcal{G} \subset \mathbf{R}^2$ is the set of *good* initial conditions. The complement of $\mathcal{G} \subset \mathbf{R}^2$ is called the *forbidden* set of equation (1) and is denoted by $\mathcal{F}$. That is,

$$\mathcal{F} = \{(x_{-1}, x_0) \in \mathbf{R}^2 : \text{Eq. (1) is not well defined for some } n \geq 0\}.$$

Our second goal is to understand the short and long term behavior of the solutions of equation (1) when $(x_{-1}, x_0) \in \mathcal{G}$.

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