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THE NORM THEOREM FOR TOTALLY SINGULAR QUADRATIC FORMS

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ABSTRACT. The aim of this paper is to prove the norm theorem in the case of totally singular quadratic forms.

1. Introduction. Let F be a commutative field. In characteristic different from 2, an important part in the algebraic theory of quadratic forms is that related to function fields of quadratic forms, and a classical result in this area is the Cassels-Pfister subform theorem. This theorem is a consequence of another one due to Knebusch and known as the norm theorem [**5**, Theorem 4.2]. It asserts that for a normed irreducible polynomial $p \in F[x_1, \dots, x_n]$, an anisotropic quadratic form ϕ becomes hyperbolic over the quotient field of $F[x_1, \dots, x_n]/(p)$ if and only if p is a norm of ϕ over $F(x_1, \dots, x_n)$ the field of rational functions in n variables $x_1, \dots, x_n$ over F (normed means that the coefficient of the highest monomial occurring in p with respect to the lexicographical ordering is 1).

For quadratic forms in characteristic 2, we should distinguish between different objects: nonsingular quadratic forms, totally singular quadratic forms, and singular but not totally singular quadratic forms, cf. subsection 2.1 for definitions, and it is an interesting problem to extend some known results in characteristic $\neq 2$ to one of these objects. The theory of function fields of nonsingular quadratic forms works as in characteristic $\neq 2$. In fact, for such quadratic forms we have the norm theorem by Baeza [**2**]. This theorem and some representation results in [**1**, Satz 3.4, 3.5, Lemma 3.7] imply the Cassels-Pfister subform theorem for nonsingular quadratic forms, which was explicitly stated in [**6**, Proposition 3.4] (recall that some representation results in [**1**] have

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