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COMPACTOIDNESS

IWO LABUDA

Dedicated to the memory of Paweł Szeptycki,
a mathematician and an artist

ABSTRACT. Although there is no doubt, today, what is the proper definition of compactness for a *subset* of a topological space X, the corresponding definition for a *family of subsets* of X is no longer evident. Two answers, arguably, are provided via the notion of *compactoidness*. The latter notion is the leitmotif of the topical survey below.

0. Introduction. The notion of compactoidness, in the form of a 'total net', can be traced back at least to 1969, see Pettis [**19**]. There, the total nets (of sets) were defined, a few facts (including a generalized version of Tikhonov product theorem) about them were proven and several interesting applications were indicated. Without any deeper analysis, essentially the same notion also made an appearance in the 1970 papers by Topsøe (as a 'compact net' [**22**]) and Wilker (as a 'compact filter' [**26**]), as well as in a 1976 paper by Kats ('compact filter' [**11**]).

It seems that it was Vaughan who first realized the importance of Pettis' contribution. He discusses the net versus filter approach in [**24**], the Proceedings of a Conference in Memphis, and gives there (some of the proofs appeared later in [**25**]) the basic characterization: *in a regular space a filter is compactoid if and only if it aims at its adherence which is compact.*

It looks as if not too many mathematicians read proceedings of conferences … . Compactoid filters, i.e., total filters of Pettis were then rediscovered again by Penot, and by Dolecki and Lechicki, see [**7, 18**]. In both cases, characteristically, the (re)discovery was motivated

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