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A NOTE ON RIESZ BASES OF EIGENVECTORS FOR A CLASS OF NONANALYTIC OPERATOR FUNCTIONS

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ABSTRACT. Riesz basis properties for a class of self-adjoint and continuous operator functions are studied. A new approach based on the spectral distribution function is presented.

1. Introduction. There is a hypothesis in the spectral theory of operator functions in the following form.

If $L(\alpha)$ is an operator function of the class $C([a,b], S(H))$ such that $L(a) \ll 0$, $L(b) \gg 0$, for all $x \in H \setminus \{0\}$, the function $(L(\alpha)x, x)$ has exactly one zero in (a,b) and $\pi(L) = \{\gamma\} \in (a,b)$, then the eigenvectors of $L(\alpha)$, corresponding to eigenvalues in (a,b) form a Riesz basis for the Hilbert space H or they are complete in H.

Here by $C([a,b], S(H))$ we denote the class of self-adjoint and continuous operator functions defined on the interval $[a,b]$, and $\pi(L)$ is the set of the limit spectrum, i.e.,

$$\pi(L) = \{\lambda \in (a,b) \mid \exists x_n,\ \|x_n\| = 1,\ x_n \to 0\,(\text{weakly}),\ L(\lambda)x_n \to 0\}.$$

The spectrum $\sigma(L)$, the point spectrum or the set of eigenvalues $\sigma_e(L)$ of L are subsets of $[a,b]$ defined as follows: $\lambda \in \sigma(L)$ if $0 \in \sigma(L(\lambda))$ and $\lambda \in \sigma_e(L)$ if $0 \in \sigma_e(L(\lambda))$. A nonzero vector x from the kernel $\ker L(\lambda)$ for $\lambda \in \sigma_e(L)$ is called an eigenvector of L corresponding to λ.

This problem in the finite-dimensional case for the class $C^1([a,b], S(H))$—the class of self-adjoint and continuously differentiable operator functions, was solved in [**1**]. For analytic operator functions the

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