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POINTS AT RATIONAL DISTANCES ON A PARABOLA

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ABSTRACT. This paper deals with the open problem of finding the maximum number of points on the parabola $y = x^2$ such that all of their mutual distances are rational. We obtain, in parametric terms, a set of seven points on this parabola such that four of the points have all of their six mutual distances rational, five of the points have all but one of their 10 mutual distances rational, six of the points have 12 mutual distances rational and the seven points have 15 of their mutual distances rational. By giving suitable numerical values to the parameters, we can obtain infinitely many examples of seven points in which the first four points, with all of their six mutual distances rational, have positive abscissae and are non-concyclic. Further, for any arbitrary positive integer n, we obtain in parametric terms the abscissae of $n + 1$ pairs of points on the given parabola such that $5n + 1$ of their mutual distances are rational. With a suitable choice of parameters, we get numerical examples with $5n+2$ of the mutual distances rational.

1. Introduction. There are several interesting diophantine problems concerning the existence, on a plane, of a set of points all of whose mutual distances are rational. For example, Guy [**5**, pp. 181–188] mentions the following open problems:

(i) Is there a point all of whose distances from the corners of the unit square are rational?

(ii) Are there more than six points in the plane, no three on a line, no four on a circle, all of whose mutual distances are rational?

This paper is concerned with an analogous open problem posed by Dean [**3**] who asks the following: "How many points can you find on the (half) parabola $y = x^2$, $x > 0$, so that the distance between any pair of them is rational?"

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