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REGULAR COMPONENTS OF MODULI SPACES OF STABLE MAPS AND K-GONAL CURVES

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ABSTRACT. Here we prove for certain integers g, rd and k the existence of a generically smooth irreducible component of the moduli space of stable maps $M_g^-(\mathbf{P}^1 \times \mathbf{P}^r, (k, d))$ with the expected dimension. As a byproduct, we obtain the existence of a generically smooth component of dimension $\rho(g, r, d) := g - (r+1)(g+r-d)$ for the Brill-Noether locus $W_d^r(C)$ of a general k-gonal curve C of genus g.

1. The statements. For any complex projective variety Y and any class $\beta \in H_2(Y, \mathbf{Z})$, one considers the moduli space $M_g^-(Y, \beta)$ of all stable maps $f : C \to Y$, with C a reduced connected nodal curve of arithmetic genus g and $f_*([C]) = \beta$ (see [**7**] for the construction of these moduli spaces). The *expected dimension* of the algebraic stack $M_g^-(Y, \beta)$ is $\dim(Y)(1-g) + 3g - 3 - b \cdot \omega_Y$. For all integers g, r, d, set $\rho(g, r, d) := g - (r+1)(g+r-d) = (r+1)d - rg - r(r+1)$ (the so-called Brill-Noether number). As in [**6**] we are interested in the case in which $Y = \mathbf{P}^1 \times \mathbf{P}^r$, and we look for irreducible components, V, of $M_g^-(\mathbf{P}^1 \times \mathbf{P}^r, \beta)$ which are *good*, i.e., such that V is generically smooth and with the expected dimension. When $Y = \mathbf{P}^1 \times \mathbf{P}^r$ the class β is given by a pair (k, d) of non-negative integers and in this case the dimension of a good component of $M_g^-(\mathbf{P}^1 \times \mathbf{P}^r, \beta)$ is $\rho(g, r, d) + 3g - 3 + 2k - g - 2$. The main aim of this paper is the proof of the following result.

Theorem 1.1. *Fix positive integers g, r, d and k such that $(g+2)/2 \geq k \geq r+3 \geq 6$, $\rho(g, r, d) \geq 0$, and $g \leq (r+1)\lfloor d/r \rfloor - r - 3$. Then there exists a good component of $M_g^-(\mathbf{P}^1 \times \mathbf{P}^r, (k, d))$.*

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