ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 3, 2006

FOURTH-ORDER SCHEMES OF EXPONENTIAL TYPE FOR SINGULARLY PERTURBED PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

A.A. SALAMA AND H.Z. ZIDAN

ABSTRACT. We present a class of difference schemes of exponential type for solving singularly perturbed parabolic partial differential equations. This class includes a scheme of fourth order of accuracy when the perturbation parameter, ε, is fixed. For small ε, the orders of accuracy are verified experimentally. Stability analysis for these schemes are also presented. Numerical results and comparisons with other schemes are considered.

1. Introduction. We consider the following parabolic partial differential equation

$$Lu(x,t) \equiv \frac{\partial u}{\partial t} - \varepsilon \frac{\partial^2 u}{\partial x^2} - b(x,t)\frac{\partial u}{\partial x} + d(x,t)u = f(x,t), \quad (x,t) \in \Omega \equiv (0,1) \times (0,T], \tag{1.1a}$$

$$u(x,0) = g(x), \quad x \in [0,1], \tag{1.1b}$$

$$u(0,t) = g_0(t), \quad u(1,t) = g_1(t), \quad \forall\, t \geq 0, \tag{1.1c}$$

where ε is a parameter in $(0,1]$. The functions g_0 and g_1 are continuous and bounded as $t \to \infty$ and b, d and f are sufficiently smooth functions of x and t. Also, we assume that $b(x,t) \geq \beta > 0$ and $d(x,t) \geq 0$ on $\overline{\Omega}$. For $\varepsilon \to 0^+$ the exact solution of equation (1.1) exhibits a boundary layer at $x = 0$. In the case of $b(x,t) \leq -\beta < 0$ the problem can be transformed to the problem (1.1) by making the change of variable $x \to 1 - x$. Problems of this type arise, for example, in the modeling

Key words and phrases. Partial differential equations, singular perturbation, higher order schemes, exponential schemes.
Received by the editors on June 26, 2003, and in revised form on December 17, 2003.