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SOME NORMAL SUBGROUPS OF THE EXTENDED HECKE GROUPS $\overline{H}(\lambda_p)$

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ABSTRACT. We consider the extended Hecke groups $\overline{H}(\lambda_p)$ generated by $T(z) = -1/z$, $S(z) = -1/(z+\lambda_p)$ and $R(z) = 1/\bar{z}$ with $\lambda_p = 2\cos(\pi/p)$ for $p \geq 3$ prime number. In this article, we study the abstract group structure of the extended Hecke groups and the power subgroups $\overline{H}^m(\lambda_p)$ of $\overline{H}(\lambda_p)$. Then, we give the relations between commutator subgroups and power subgroups and also the information of interest about free normal subgroups of the extended Hecke groups.

1. Introduction. In [**7**], Hecke introduced the groups $H(\lambda)$ generated by two linear fractional transformations

$$T(z) = -\frac{1}{z} \quad \text{and} \quad U(z) = z + \lambda,$$

where λ is a fixed positive real number. Let $S = TU$, i.e.,

$$S(z) = -\frac{1}{z+\lambda}.$$

Hecke showed that H(λ) is discrete if and only if $\lambda = \lambda_q = 2\cos(\pi/q)$, $q \in \mathbf{N}$, $q \geq 3$, or $\lambda \geq 2$. We will focus on the discrete with $\lambda < 2$, i.e., those with $\lambda = \lambda_q$, $q \geq 3$. These groups have come to be known as the *Hecke groups*, and we will denote $H(\lambda_q)$ for $q \geq 3$. Hecke group $H(\lambda_q)$ is isomorphic to the free product of two finite cyclic groups of orders 2 and q, and it has a presentation

$$H(\lambda_q) = \langle T, S \mid T^2 = S^q = I \rangle \cong C_2 * C_q.$$

The first several of these groups are $H(\lambda_3) = \Gamma = PSL(2, \mathbf{Z})$, the modular group, $H(\lambda_4) = H(\sqrt{2})$, $H(\lambda_5) = H((1+\sqrt{5})/2)$, and $H(\lambda_6) = H(\sqrt{3})$. It is clear that $H(\lambda_q) \subset PSL(2, \mathbf{Z}[\lambda_q])$, for $q \geq 4$.

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