

## CHARACTERIZATIONS OF GENERALIZED NEVAI'S CLASS AT THE BOUNDARY POINTS OF CONTRACTED ZEROS

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ABSTRACT. For generalized Nevai's class, the ratio asymptotics of orthogonal polynomials are obtained by many authors outside the contracted zero interval. We prove that the asymptotic properties can be extended to the point which is essentially a boundary point of the contracted zeros by the coefficients of three term recurrence relation and then several characterizations are found. Lastly we give several applications for such characterizations.

**1. Introduction.** Let  $\mu$  be a positive measure with infinitely many points of increase and all moments are finite. The corresponding orthonormal polynomials  $\{P_n(x)\}_{n=0}^{\infty}$  such that

$$\int_{-\infty}^{\infty} P_m(x)P_n(x) d\mu(x) = \delta_{mn}, \quad m, n \geq 0,$$

where  $\delta_{mn}$  is the Kronecker delta, satisfy a three term recurrence relation

$$\begin{aligned} xP_n(x) &= a_{n+1}P_{n+1}(x) + b_nP_n(x) + a_nP_{n-1}(x), \\ P_0(x) &= 1, \quad P_{-1} = 0, \\ n &= 0, 1, 2, \dots, \end{aligned}$$

where the coefficients are uniquely determined by

$$a_n = \int_{-\infty}^{\infty} xP_n(x)P_{n-1}(x) d\mu(x) > 0, \quad n = 1, 2, \dots,$$

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