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MIXED ORDER SYSTEMS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT. Expansions into eigenfunctions and associated functions of nth order ordinary scalar differential equations and first order systems of ordinary linear differential equations have been extensively investigated. Here we consider systems of higher order differential equations. In this case, it is not always possible to obtain an associated first order systems. Two mixed order systems which are equivalent to first order systems are considered. Results on eigenfunction expansion are established for each of them.

1. Introduction. Expansions into eigenfunctions and associated functions of nth order ordinary scalar differential equations and first order systems of ordinary linear differential equations have been extensively investigated. However, it seems that there are no such results for mixed order systems of differential equations. When dealing with nth order differential equations or systems of first order equations on a bounded interval, the assumption of suitably regular coefficients guarantees that the spectrum is either all of $\mathbf{C}$ or consists of isolated eigenvalues. One obstacle to eigenfunction expansion for mixed order systems is that this alternative need not be true for mixed order systems, see e.g., [**3**]. The problem here is that a mixed order system of differential equations cannot always be transformed into a first order system of differential equations. We will henceforth focus our attention on the case that the mixed order system has constant coefficients and is equivalent to a first order system

$$y' = A(\lambda)y, \tag{1.1}$$

where λ is the eigenvalue parameter.

We are going to use estimates of the resolvent of the operator associated with (1.1) and suitable boundary conditions as obtained

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