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POINT X-RAYS OF CONVEX BODIES IN PLANES OF CONSTANT CURVATURE

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ABSTRACT. Let K be a convex body in a plane of constant curvature $\mathcal{R}^2$. The X-ray of K at a point $p \in \mathcal{R}^2$ gives the lengths of all the intersections of K with geodetics through p. Hammer's X-ray problem queries how many points must be taken to permit the exact reconstruction of K from the corresponding X-rays. The known answer in the Euclidean plane is here extended by proving that a convex body in $\mathcal{R}^2$ is distinguished from others by its X-rays from four point sources in general position, up to a reflection in the origin on the sphere. As a further generalization, we prove that three points in general position suffice when we work with the class of spherical lunes.

1. Introduction. In this paper we continue the discussion of Hammer's X-ray problem in non-Euclidean spaces, which was begun in [**2**] and continued in [**3**].

Our setting here is a complete simply connected Riemannian surface of constant curvature k which is, as is well known, either a Euclidean plane E^2, $k = 0$, or sphere S_k^2, $k > 0$, or hyperbolic plane H_k^2, $k < 0$. We shall scale the distance function induced by the Riemannian metric so that $k = 0, 1, -1$ and denote the spaces E^2, $S^2 := S_1^2$ and $H^2 := H_{-1}^2$ simply by $\mathcal{R}^2$.

Let $K \subset \mathcal{R}^2$ be a compact convex set with nonempty interior. The X-ray of K at a point $p \in \mathcal{R}^2$ gives the lengths of all the intersections of K with lines through p. Hammer's X-ray problem [**10**] (and its non-Euclidean generalization) queries how many points must be taken to permit the exact reconstruction of K from the corresponding X-rays.

In E^2 the answer is due to Volčič [**16**], who proved that four points, no three collinear, are enough to distinguish K among all other planar convex bodies. He was the first who employed the idea of using

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