

**BOUNDARY VALUE PROBLEM FOR
SECOND-ORDER DIFFERENTIAL OPERATORS
WITH NONREGULAR INTEGRAL
BOUNDARY CONDITIONS**

M. DENCHE AND A. KOURTA

ABSTRACT. In this paper, we study a nonregular second order differential operator with weighted integral boundary conditions. Under certain conditions on the weighting functions which occur in the integral boundary conditions expressed in terms of the values at the interval endpoints, we prove that the resolvent has no minimal growth. Furthermore, the studied operator generates an analytic semi-group with singularities in $L^1(0, 1)$. The obtained results are then used to show the correct solvability of a mixed problem for a parabolic partial differential equation with nonregular integral boundary conditions.

1. Introduction. In space $L^1(0, 1)$, we consider the boundary value problem

$$(1) \quad \begin{cases} L(u) = u''(x) = f(x), \\ B_i(u) = \int_0^1 R_i(t)u(t) dt + \int_0^1 S_i(t)u'(t) dt = 0 \quad i = 1, 2, \end{cases}$$

where functions $R_i, S_i \in C([0, 1], \mathbf{C})$, $i = 1, 2$. We associate to problem (1) in space $L^1(0, 1)$ the operator

$$L_1(u) = u'',$$

with domain $D(L_1) = \{u \in W^{2,1}(0, 1) : B_i(u) = 0, i = 1, 2\}$.

Many papers and books give the full spectral theory of Birkhoff regular differential operators with two point linearly independent boundary conditions, in terms of coefficients of boundary conditions. The reader should refer to [7, 11, 21–23, 30, 33, 35] and references therein. Few works were devoted to the study of a nonregular situation. The

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