ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 4, 2006

GENERALIZED RADON TRANSFORM

R. ROOPKUMAR

ABSTRACT. We extend the Radon transform to the context of Boehmians consistent with the Radon transform on the space of tempered distributions and on the space of distributions by constructing suitable Boehmian spaces. We also prove that the generalized Radon transforms are continuous, linear, bijections and their inverses are also continuous.

1. Introduction. Starting from the work [**13**] of Radon, the theory of Radon transform has been developed on various testing function spaces and distribution spaces, and their properties have been discussed. See [**1**–**6**]. Let $\mathcal{S}(\mathbf{R}^N)$ denote the space of smooth rapidly decreasing functions on $\mathbf{R}^N$, $N > 1$, and $\mathcal{D}(\mathbf{R}^N)$ denote the space of smooth functions with compact supports. Their dual spaces $\mathcal{S}'(\mathbf{R}^N)$, $\mathcal{D}'(\mathbf{R}^N)$ are called the space of tempered distributions and the space of distributions respectively. We also denote by $\mathcal{I}(\mathbf{R}^N)$, the space of all slowly increasing continuous functions on $\mathbf{R}^N$ with the following notion of convergence: $\eta_n \to \eta$ as $n \to \infty$ if there exists $m \in \mathbf{N}$ and a sequence (C_n) in $\mathbf{R}$ converging to zero, such that $|\eta_n(x) - \eta(x)| \leq C_n(1 + \|x\|^m)$, for all $x \in \mathbf{R}^N$, where $\|x\|$ is the Euclidean norm of $x \in \mathbf{R}^N$.

The space $R\mathcal{S}$ consists of all functions $f\colon \mathbf{R} \times S^{N-1} \to \mathbf{C}$ satisfying the following conditions:

$$\alpha : \|f\|_m = \sup_{\substack{(s,w)\in\mathbf{R}\times S^{N-1}\\ 0\leq k\leq m}} (1+s^2)^m \left|\frac{\partial^k}{\partial s^k} f(s,w)\right| < \infty.$$

$\beta : f(s,w) = f(-s,-w)$ for all $s \in \mathbf{R}$ and $w \in S^{N-1}$.

$\gamma : \displaystyle\int_{-\infty}^{\infty} f(s,w) s^k \, ds$ is a polynomial of degree $\leq k$ in w, $\forall\, k \in N_0$.

2000 AMS *Mathematics Subject Classification.* Primary 44A12, 44A90, Secondary 40F12.
Key words and phrases. Boehmians, convolution, distributions, Radon transforms.
Received by the editors on November 11, 2003, and in revised form on February 20, 2004.