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THE VALUES OF ADDITIVE FORMS AT PRIME ARGUMENTS

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ABSTRACT. New results are proved on additive forms at prime arguments of the type $\lambda_1 p_1^k + \cdots \lambda_s p_s^k$ where the λ_j are not all negative and are not all in rational ratio. The improvements come in the number of variables required and the distribution of the values. The former improvement comes from using familiar techniques in the Hardy-Littlewood method, while the latter improvement stems from recent developments in the theory of exponential sums.

1. Introduction. Given $k \geq 1$ and s nonzero real numbers $\lambda_1, \dots, \lambda_s$ (not all in rational ratio, not all negative), we write

$$F(\mathbf{p}) = \sum_{j=1}^{s} \lambda_j p_j^k$$

where $\mathbf{p} = (p_1, \dots, p_s)$ with each p_j a prime. Various authors have considered the distribution of values of such forms, for example, see [**14**]. Here we continue in the direction started by Brüdern, Cook and Perelli [**3**] and followed by Cook and Fox [**5**], Cook [**4**] and Harman [**9**]. We call a set of positive reals $\mathcal{V}$ a *well-spaced set* if there is a $c > 0$ such that

$$u, v \in \mathcal{V}, \quad u \neq v, \quad \Longrightarrow |u - v| > c.$$

In order to get the full-strength of the results under consideration, one should also assume that

$$|\{v \in \mathcal{V} : v \leq X\}| \gg X^{1-\varepsilon},$$

though our results are nontrivial with a weaker lower bound. Given a form F as above, let $E_k(\mathcal{V}, X, \delta)$ denote the number of $v \in \mathcal{V}$, $v \leq X$, such that the inequality

$$|F(\mathbf{p}) - v| < v^{-\delta} \tag{1.1}$$

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