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A SIMPLE PROOF THAT A LINEARLY ORDERED SPACE IS HEREDITARILY AND COMPLETELY COLLECTIONWISE NORMAL

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It is known [**1**] that a linearly ordered space is hereditarily collectionwise normal. In this note we give a simpler proof that a linearly ordered space is both hereditarily and completely collectionwise normal [**3**, p. 168].

Let X be a linearly ordered set endowed with the usual open interval topology. We denote intervals in the usual way by (a, b), $(a, b]$, $[a, b)$ or $[a, b]$. We prove

Theorem I. *Let $\{A_i\}$ be a family of subsets of X such that each A_i is disjoint from the closure of $\cup_{j \neq i} A_j$. Then there is a family $\{U_i\}$ of mutually disjoint open sets such that $A_i \subset U_i$ for each index i.*

Proof. For convenience, put $P = \cup_i A_i$. We say that points $a, b \in X \backslash P$ are equivalent if the interval joining a to b is a subset of $X \backslash P$. Then $X \backslash P$ is partitioned into equivalence classes we call the *components* of $X \backslash P$. Use the Axiom of Choice to select a point $f(C)$ in each component C.

Fix an index i. For each $x \in A_i$ that is not the greatest point in X we select a point $t_x > x$ as follows:

Case (1). If x is a right accumulation point of A_i, select $t_x \in A_i$ so that $t_x > x$ and the interval (x, t_x) is disjoint from $P \backslash A_i$.

Case (2). If x has an immediate successor, we designate it by t_x.

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