ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 4, 2006

UNITARY GROUPS ACTING ON GRASSMANNIANS ASSOCIATED WITH A QUADRATIC EXTENSION OF FIELDS

CLAUDIO G. BARTOLONE AND M. ALESSANDRA VACCARO

ABSTRACT. Let (V, H) be an anisotropic Hermitian space of finite dimension over the algebraic closure of a real closed field K. We determine the orbits of the group of isometries of (V, H) in the set of K-subspaces of V.

Throughout the paper K denotes a real closed field and $\overline{K}$ its algebraic closure. Then it is well known (see, for example, [**4**, Chapter 2], [**23**]; see also [**8**]) that $\overline{K} = K(i)$ with $i = \sqrt{-1}$. Also we let (V, H) be an anisotropic Hermitian space (with respect to the involution underlying the quadratic field extension $\overline{K}/K$) of finite dimension n over $\overline{K}$. In this context we consider the natural action of the *unitary* group $U = U(V, H)$ of isometries of (V, H) on the set X_d of all d-dimensional K-subspaces of V. The analogous problem where (V, H) is a symplectic space was treated in [**1**] (for arbitrary quadratic field extensions). It turns out that, in contrast with the symplectic case, there are infinitely many orbits for the action of the unitary group U on X_d.

In group theoretic language the stated problem turns into the determination of the double coset spaces of the form

$$G_W \setminus G \,/\, U, \tag{1}$$

where $G = \mathrm{GL}\,(V_K)$ and G_W denotes the parabolic subgroup of G stabilizing a member $W \in X_d$ (we write V_K to indicate that we are regarding V as a vector space over K). The precise structure of double coset spaces involving classical groups is of great interest in applying the classical Rankin-Selberg method for explicit construction of automorphic L-functions, as Garrett [**2**] and Piatetski-Shapiro and Rallis [**6**] worked out.

2000 AMS *Mathematics Subject Classification.* Primary 51N30, 15A21, Secondary 11E39.
Received by the editors on October 13, 2003.